Definitions and key facts for section 3.2

Fact: The determinant and row operations
Let A be a square matrix.

1. If a multiple of one row of A is added to another row to product a matrix B, then $\det B = \det A$.
2. If two rows of A are interchanged to produce B, then $\det b = -\det A$.
3. If one row of A is multiplied by k to produce B, then $\det B = k \cdot \det A$.

If A is reduced to an echelon form U using only row replacment and row interchange operations, then

$$\det A = \begin{cases} (-1)^r \cdot (\text{product of pivots in } U) & \text{when } A \text{ is invertible} \\ 0 & \text{when } A \text{ is not invertible} \end{cases}$$

From this we obtain one more item in the invertible matrix theorem.
Fact: A square matrix A is invertible if and only if $\det A \neq 0$.

Fact: Further properties of the determinant
Let A and B be $n \times n$ matrices, then

1. $\det A^T = \det A$, and
2. $\det AB = (\det A)(\det B)$